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A note on some non-linear covariant gauges in Yang-Mills theory.

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ABSTRACT

We study a class of non-linear covariant gauges containing only one gauge parameter. This class of gauges can be characterized by requiring that the gauge-fixing action, in addition to the BRS symmetry, is invariant under a second global symmetry which commutes with the BRS symmetry, and which leads to a Ward identity which is the analogue of the equation of motion of the antighost in the case of the linear gauges and which ensures the stability of the model under radiative corrections.

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1. Introduction.

The general non-linear covariant gauge of Yang-Mills theory has been studied in a recent paper [1] where the renormalization of the theory is controlled simply by the Slavnov-Taylor identity. In the case of a simple group, one has to introduce three gauge constants which are independently renormalized. If one wishes to work with a smaller number of gauge constants, one has to use auxiliary constraints to ensure the stability of the gauge-fixing action. For instance, the stability of linear gauges is ensured by the antighost equation of motion [2]. In the non-linear case, one may use the anti-BRS [3] symmetry which allows only for two gauge constants.

Here we wish to describe a class of non-linear gauges, already proposed by Delbourgo and Jarvis [4], containing only one gauge constant.

The interest in this class of gauges is due to some peculiar features which allow us to formulate the model in complete analogy with the linear case; indeed, this class of gauges can be characterized by imposing, in addition to the BRS symmetry, a second global symmetry which commutes with the BRS symmetry.

This symmetry leads to a Ward identity which plays a role analogous to the antighost equation of motion in linear gauges and which guarantees the stability of the model under radiative corrections.

It is interesting to observe that this class of gauges naturally appears in the massive models considered by Curci and Ferrari [3] and by Ojima [5].

2. Classical action.

We start from a pure Yang-Mills action with simple gauge group G

$$S_{inv} = -\int d^4x \, \frac{1}{4g^2} F^{a\mu\nu} F^a_{\mu\nu} \, . \tag{2.1}$$

The gauge-fixed action will be required to be invariant under BRS transformations [2]:

$$\begin{aligned} sA^a_\mu &= -(D_\mu c)^a \qquad & s\bar{c}^a &= b^a \\ sc^a &= \frac{1}{2} f^{abc} c^b c^c \qquad & sb^a &= 0 \, , \end{aligned} \tag{2.2}$$

and under the following global symmetry:

$$\delta\bar{c}^a = c^a \qquad \delta b^a = \frac{1}{2} f^{abc} c^b c^c \qquad \delta A^{a\mu} = \delta c^a = 0 \, , \tag{2.3}$$

which commutes with BRS transformations

$$[s, \delta] = 0 \, . \tag{2.4}$$

In (2.2), (2.3) $c^a, \bar{c}^a$ and b^a are respectively the ghost, the antighost and the Lagrange multiplier, and

$$(D_\mu c)^a = \partial_\mu c^a + f^{abc} A^b_\mu c^c \, . \tag{2.5}$$

Transformations (2.3) correspond to the generator of the $SL(2, R)$ symmetry appearing in [4] which commutes with the BRS symmetry.

We then choose as gauge-fixing action

$$\begin{aligned} S_{gf} &= \int d^4x \; s\Big(\bar{c}^a(\partial_\mu A^{a\mu}) + \bar{c}^a \frac{\alpha}{2}\big(b^a - \frac{1}{2} f^{abc} \bar{c}^b c^c\big) \Big) \\ &= \int d^4x \Big(\; b^a(\partial_\mu A^{a\mu}) + \bar{c}^a \partial^\mu (D_\mu c)^a + \frac{\alpha}{2} b^a b^a - \frac{\alpha}{2} f^{abc} b^a \bar{c}^b c^c - \frac{\alpha}{8} f^{abc} f^{cmn} \bar{c}^a \bar{c}^b c^m c^n \; \Big) \, , \end{aligned} \tag{2.6}$$

which is easily seen to be invariant under (2.2) and (2.3). To write down the Ward identity corresponding to (2.2) and (2.3), we couple the non-linear variations to external sources:

$$S_s = \int d^4x \Big(\; -\Omega^{a\mu}(D_\mu c)^a + L^a \frac{f^{abc}}{2} c^b c^c \; \Big) \, . \tag{2.7}$$

Then the complete classical action

$$\Sigma = S_{inv} + S_{gf} + S_s \, , \tag{2.8}$$

satisfies the Slavnov-Taylor identity

$$\int d^4x \left(\frac{\delta\Sigma}{\delta\Omega^{a\mu}} \frac{\delta\Sigma}{\delta A^a_\mu} + \frac{\delta\Sigma}{\delta L^a} \frac{\delta\Sigma}{\delta c^a} + b^a \frac{\delta\Sigma}{\delta\bar{c}^a} \right) = 0 \, , \tag{2.9}$$

and the Ward identity

$$\int d^4x \left(c^a \frac{\delta\Sigma}{\delta\bar{c}^a} + \frac{\delta\Sigma}{\delta b^a} \frac{\delta\Sigma}{\delta L^a} \right) = 0 \, . \tag{2.10}$$

The mass dimensions of the fields $A^a_\mu, b^a, c^a, \bar{c}^a, L^a, \Omega^{a\mu}$ are respectively 1, 2, 0, 2, 4, 3 and the assigned ghost charges 0, 0, 1, -1, -2, -1.

The Ward identity (2.10) can be seen as the analogue of the standard equation of motion of the antighost in the case of the linear gauges and, as we will see in the next section, it will ensure the stability of the model under radiative corrections.

Before studying the stability of (2.9), (2.10) let us relate our model to the massive models considered in [3,5]. There a mass term is added to the classical action:

$$S_m = \int d^4x \left(\frac{m^2}{2} A^{a\mu} A^a_\mu + \alpha m^2 \bar{c}^a c^a \right) . \tag{2.11}$$

The action $S_{inv} + S_{gf} + S_m$ is invariant under a modified BRS transformation, which reads:

$$\begin{aligned} s_m A^a_\mu &= -(D_\mu c)^a & s_m \bar{c}^a &= b^a \\ s_m c^a &= \frac{1}{2} f^{abc} c^b c^c & s_m b^a &= -m^2 c^a . \end{aligned} \tag{2.12}$$

This symmetry is no longer nilpotent, but satisfies instead:

$$s_m^2 = -m^2 \delta . \tag{2.13}$$

Thus in the massive case, the invariance (2.3) is a consequence of the modified BRS symmetry. Notice finally that, both in the massive and massless cases, there is also an anti-BRS symmetry $\bar{s}$ obtained from s by the replacements:

$$\begin{aligned} c^a &\to \bar{c}^a & \bar{c}^a &\to -c^a \\ A^{a\mu} &\to A^{a\mu} & b^a &\to b^a - f^{abc} \bar{c}^b c^c , \end{aligned} \tag{2.14}$$

and, correspondingly, a new commuting invariance $\bar{\delta}$ which, in the massive case, appears through:

$$\bar{s}_m^2 = -m^2 \bar{\delta} . \tag{2.15}$$

These generators form with the Faddeev-Popov charge Q_{FP} a superalgebra $osp(1,2)$:

$$\begin{aligned} \{s_m, \bar{s}_m\} &= m^2 Q_{FP} & \{\delta, \bar{\delta}\} &= -Q_{FP} \\ \{s_m, \bar{\delta}\} &= \bar{s}_m & \{\bar{s}_m, \delta\} &= -s_m . \end{aligned} \tag{2.16}$$

3. Stability.

We come back to the massless case and look for the most general local functional which satisfies eqs. (2.9), (2.10). We limit ourselves to perturbations of the classical action Σ, which are of dimensions 4, Faddeev-Popov charge 0 and are invariant under global transformations of the gauge group. The perturbation $\tilde{\Sigma}$ satisfies the linearized equations:

$$\begin{aligned} D_\Sigma \tilde{\Sigma} &= 0 \\ W_L \tilde{\Sigma} &= 0 , \end{aligned} \tag{3.1}$$

where

$$D_\Sigma = \int d^4x \left(\frac{\delta\Sigma}{\delta\Omega^{a\mu}} \frac{\delta}{\delta A^a_\mu} + \frac{\delta\Sigma}{\delta A^{a\mu}} \frac{\delta}{\delta\Omega^a_\mu} + \frac{\delta\Sigma}{\delta L^a} \frac{\delta}{\delta c^a} + \frac{\delta\Sigma}{\delta c^a} \frac{\delta}{\delta L^a} + b^a \frac{\delta}{\delta \bar{c}^a} \right) , \tag{3.2}$$

$$W_L = \int d^4x \left(c^a \frac{\delta}{\delta \bar{c}^a} + \frac{\delta\Sigma}{\delta b^a} \frac{\delta}{\delta L^a} + \frac{\delta\Sigma}{\delta L^a} \frac{\delta}{\delta b^a} \right) . \tag{3.3}$$

are commuting operators.

The solution to eq. (3.2) is well known [1,2] and may be written as:

$$\begin{aligned} \tilde{\Sigma} = -2z_g S_{inv} + D_\Sigma \int d^4x \Big(& z_A \Omega^a_\mu A^{a\mu} - z_c L^a c^a + z_{\bar{c}} \bar{c}^a \partial_\mu A^{a\mu} \\ & + \frac{\alpha}{2}(z_\alpha + 2z_{\bar{c}}) \bar{c}^a (b^a - \frac{1}{2} f^{abc} \bar{c}^b c^c) + \lambda_{abc} \bar{c}^a \bar{c}^b c^c + \mu_{abc} \bar{c}^a A^{b\mu} A^c_\mu \Big) \end{aligned} \tag{3.4}$$

where $z_g, z_A, z_c, z_{\bar{c}}, z_\alpha$ are constants and λ_{abc}, μ_{abc} are invariant tensors. All terms but the last 2 may be reabsorbed by multiplicative renormalization of the fields and coupling constants. Using finally (3.3) we obtain:

$$\lambda_{abc} = 0 \quad , \quad \mu_{abc} = 0 . \tag{3.5}$$

Thus the model described by the action (2.6) will, in the absence of gauge anomalies, be multiplicative renormalizable.

Notice that the same study in the massive case leads to multiplicative renormalizability, with the same renormalization constants. In particular, the mass is not renormalized.

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